DESIGN OF TWO WHEELED ELECTRIC VEHICLE

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

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AYÇA GÖÇMEN

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MASTER OF SCIENCE

IN

THE DEPARTMENT OF MECHATRONICS ENGINEERING

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Approval of the Graduate School of Natural and Applied Sciences, Atılım University.

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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ABSTRACT

DESIGN OF TWO WHEELED ELECTRIC VEHICLE

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Two wheeled self balancing electric vehicle is studied in this thesis. The system, 2TEA (2 Tekerlekli Elektrikli Araç – 2 Wheeled Electric Vehicle) is able to operate in transporter mode and robotic mode. The first goal is to maintain stabilization in pitch dynamics. This thesis focuses on designing and implementing a state feedback controller to stabilize system on transporter mode. The system moves forward (or backward) when the driver leans forward (or backward) in transporter mode in order to stabilize body. Also, observer design is implemented on robotic mode. Thus, velocity is fed back to the system. In addition, this study covers physical improvement, parameter calculations and mathematical model improvement.

Keywords: Two wheeled electric vehicle, robotic system, stabilization, state feedback control

ÖZ

İKİ TEKERLEKLİ ELEKTRİKLİ ARAÇ TASARIMI

Göçmen, Ayça

Yüksek Lisans, Mekatronik Mühendisliği Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Kutluk Bilge Arıkan

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Bu tezde, iki tekerlekli kendini dengeleyebilen elektrikli araç çalışıldı. 2TEA (2 Tekerlekli Elektrikli Araç) hem taşıyıcı hem de robotik modda çalışabilmektedir. İlk hedef yunuslama dinamiğinde kararlılığı sağlamaktır. Bu tez, durum geri beslemeli kontrol metotunun tasarımı ve uygulanmasına yoğunlaşmaktadır. Sistem taşıyıcı modundayken dengeyi sağlayabilmek için sürücü öne(veya arkaya) eğildiğinde öne(veya arkaya) doğru hareket etmektedir. Ayrıca gözlemci tasarımı robotik durum için uygulanmaktadır. Böylelikle hız durumu sisteme geri besleme olarak dönebilmektedir. Bunlara ek olarak, bu çalışma fiziksel sistemin iyileştirilmesi, parametre hesabı ve matematiksel modelin iyileştirilmesini kapsamaktadır.

Anahtar Kelimeler: İki tekerlekli elektrikli araç, robotik sistem, kararlılık, durum geri beslemeli kontrol

To My Mother, Father and Sister

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LIST OF ABBREVIATIONS

A/D	-	Analog-to-Digital
AHRS	-	Attitude and Heading Reference System
COG	-	Center of Gravity
CPU	-	Central Processing Unit
DC	-	Direct Current
DSP	-	Digital Signal Processing
FBD	-	Free Body Diagram
FPGA	-	Field Programmable Gate Array
EMF	-	Electromotive Force
GPS	-	Global Positioning System
HT	-	Human Transporter
IMU	-	Inertial Measurement Unit
IR	-	Motor Current – Motor Resistance
Li-Po	-	Lithium - Ion Polymer
LQR	-	Linear Quadratic Regulator
MECE	-	Mechatronics Engineering
PC	-	Personal Computer
PID	-	Proportional – Integral – Derivative
PD	-	Proportional – Derivative

PWM	-	Pulse Width Modulation
RMP	-	Robotic Mobility Platform
RTWT	-	Real Time Windows Target
2TEA	-	2 Tekerlekli Elektrikli Araç

NOMENCLATURE

ϕ	-	Angular position of body
$\dot{\phi}$	-	Angular velocity of body
$\ddot{\phi}$	-	Angular acceleration of body
θ	-	Angular position of wheel
$\dot{\theta}$	-	Angular velocity of wheel
$\ddot{\theta}$	-	Angular acceleration of wheel
x	-	Linear displacement in x direction
$\dot{x}$	-	Linear velocity in x direction
$\ddot{x}$	-	Linear acceleration in x direction
x_b	-	Linear displacement of body in x direction
$\dot{x}_b$	-	Linear velocity of body in x direction
$\ddot{x}_b$	-	Linear acceleration of body in x direction
y_b	-	Linear displacement of body in y direction
$\dot{y}_b$	-	Linear velocity of body in y direction
$\ddot{y}_b$	-	Linear acceleration of body in y direction
m	-	Mass of wheel
M	-	Mass of body
I_b	-	Moment of inertia of body

I_w	-	Moment of inertia of wheel
r	-	Wheel radius
g	-	Gravitational acceleration
α	-	Acceleration
b	-	Friction coefficient
L	-	Length of body to center of gravity
F_x	-	Force in x direction
F_y	-	Force in y direction
F_t	-	Traction force
N	-	Normal force
T	-	Torque
T_f	-	Frictional torque
n	-	Gear ratio
T_m	-	DC Motor torque
θ_m	-	Angular position of motor shaft
$\dot{\theta}_m$	-	Angular velocity of motor shaft
K_e	-	Back EMF constant
K_t	-	Torque constant
R	-	Motor terminal resistance
l	-	Motor inductance
i	-	Motor armature current
V	-	Voltage applied
V_e	-	Terminal voltage

L_i	-	Length of rope in inertia test setup
R_i	-	Hanging distance from rotation axis in inertia test setup
m_i	-	Mass of system in inertia test setup
T_n	-	Period in inertia test setup

CHAPTER 1

INTRODUCTION

In the early 2000s, two-wheeled self-balancing electric vehicles took part in literature. They have been popular as a human transporter in the automotive field and a significant system in robotic applications until today. Their stability problem and their design as smart-electric vehicle make the system interesting in academic environments. Many studies about this system are in progress in computer, electrical, electronics, mechanical and mechatronics engineering branches in the universities. This two-wheeled system is explained as an interaction between robotics technologies and automotive. In the future, two-wheeled systems will commonly take part in daily life as a human transporter. These electric vehicles will be utilized in the factories, shopping malls, airports, urban transportation and similar environments. Their high maneuverability, zero turning radius, fast response, dimensional features and robotic properties make these systems usable in the narrow areas.

Smart controllers and sensor systems in robotic technologies are widely used in land vehicles. The technology transfer from robotics to vehicles increases. Popularity of the electric vehicle encourages new designs inspired by robots. Therefore, concentrating on designs and researches about electric vehicles become important.

2TEA is a robotic transporter which has the ability to work as manned and unmanned. It is an electric vehicle which is able to carry drivers in various moments of inertia. Also, it is an introduction of robotic platform which will be used for load carrier on unmanned mode in the future.

1.1 Aim and Scope of Thesis

2TEA is a two wheeled electric vehicle, which is designed and manufactured during the MECE 451 Mechatronics in Automotive Engineering course in Fall 2009. Two wheeled systems are utilized in various forms within the studies of Flying Robotics Laboratory and the Mechatronic Systems Laboratory of the Department of Mechatronics Engineering. Among these applications, 2TEA is the one that can be used for both manned and unmanned operations. Depending on these operations, followings reveal the aim and scope of this thesis.

- *Improvement of the physical system*

Structural analysis of the system is achieved during the design phase with finite element method. Structural modifications are not considered within this thesis. Inertial measurement unit is placed in a more suitable place inside the box-like structure. Two different encoder units are placed onto the system; one on the side wall of the structure and the other one placed onto the output shaft of the dc motor. Cabling and interior placement of the single board computer, battery pack are rearranged. In front of 2TEA, a carrier and load holder unit is mounted for manned operations. In future studies, this simple load carrier unit will also be utilized in unmanned operations designing controllers to reject the disturbing pitch moment due to carried load. It has 0.6 m stroke in vertical axis.

- *Calculation of the physical parameters and improvement of the mathematical model*

Pitch and translational dynamics of the vehicle are studied in this thesis. Rotational – yaw- dynamics is excluded. In physical application, when rotational commands are received to the control computer –a single board computer- additional voltage offset is introduced to the motors. Once the system is stabilized in pitch dynamics yaw motion is easy to control. Therefore, coupled pitch and translational dynamics are modeled only. Mathematical model is derived using the equations of motion, namely 2 nonlinear second order differential equations. Linearized state-space model is utilized to design controllers and the observer. Numerical values of physical parameters such as back EMF and torque constants of DC motors, pitch moment of inertia of the vehicle, etc. are required to realize the model structure. In this study, experiments that are performed to calculate the motor parameters and moment of inertia of the system are presented.

- *Design and implementation of simple and low cost state feedback controllers for manned and unmanned operations*
2TEA is considered for different applications. Transporter mode is the manned operation mode to carry a driver with the vehicle. In this mode, driver steers the vehicle while the control system maintains the stability in pitch dynamics autonomously. Basic sensor in transporter mode control is the inertial measurement unit. It is desired to have low computational cost during real time operation. Maintaining the stability in pitch dynamics is the major criteria.

In transporter mode, forward and backward motions, i.e. motion in x-direction are ignited by body lean. Driver on the vehicle leans his/her body towards forward (backward) and the control system runs the vehicle forward (backward) to maintain the stability. As the amount of lean increases forward/backward velocity also tends to increase. In simulations, body lean is modeled as an additional external torque input about pitch axis in the state-space model. As the pilot starts leaning, the amount of external torque increases up to a constant value and as the pilot reduces the amount of lean, the external torque decreases and as the pilot stands in an upright position on the vehicle the external lean torque is equal to zero. Simple regulators are designed and implemented on the vehicle to maintain the abovementioned performance. Designed regulator should satisfy robust stability because various people with different inertial properties may use the vehicle. Robust controller designs such as H_{inf} are not covered in this thesis. Rather, the robustness of the designed controllers is examined on the physical system with various drivers. Designing robust controllers based on optimization techniques will be studied in future projects and graduate studies.

The second operation mode for 2TEA is unmanned –robotic vehicle- mode. It is desired to use 2TEA for various applications as a robotic platform. Therefore, specific controllers should be designed for this mode. In the scope of this thesis, unmanned mode is only considered as the operation of 2TEA without a driver. As a robotic vehicle, 2TEA will be equipped with additional sensors besides inertial measurement unit and encoder. Control of the system with additional sensors such as GPS will be studied in future projects. Also, the stability should be maintained while reference velocity inputs will be tracked by 2TEA. Load holder and carrier unit is not considered for the unmanned mode. It means no disturbing pitch moment is considered during the controller design for the unmanned mode in this thesis. However, in future studies, load carrying will be discussed and robust controllers will be designed to maintain the robust stability and performance in use of such manipulators.

1.2 Outline of the Thesis

The literature survey is mentioned in Chapter 2. Physical system is described and the components are introduced in Chapter 3. Mathematical modeling is the essential issue to design a controller and simulate the system. It is explained under Chapter 4. After mathematical model is derived, control system design and simulations are performed. It is explained in Chapter 5. Designed control system is implemented on real system and these experiments are in Chapter 6. In the end, discussion and conclusion of thesis are stated in Chapter 7.

CHAPTER 2

LITERATURE SURVEY

Two-wheeled, self balancing systems are studied in many different concepts. They can be considered as robotic platform or as electric vehicle/transporter. Researchers focus on various issues besides the main problem stability.

Segway Human Transporter (HT), which is invented by Dean Kamen, is known as the first two-wheeled, self-balancing system in the literature. Flexibility, safety and performance are important due to being commercial product. Segway HT is demonstrated in Figure 2.1. Also, Segway brings out two wheeled self balancing robotic platform which is called Robotics Mobility Platform (RMP).

Figure 2.1 - Segway HT, [36]

Two-wheeled self-balancing systems are mainly classified into two groups as robotic platforms and transporters according to their structure. Robotic platforms are generally constructed as small sized [1-9]. Figure 2.2 is that kind of robot. However, some studies do not consider the size [10-16], as in Figure 2.3. Moreover, human-scaled robots exist [17]. Some of them are driven by an operator while rests are driven autonomously. Operator controlled robots are moved by remote control as in [1, 2, 6, 9, 10, 13, 18, 19, 20]. This can be achieved by receiving command from a personal computer (PC) via a bluetooth module or a radio receiver. Fully autonomous robots may also have intelligence in [17, 21, 22]. These robots generally have camera in order to detect the environment and path planning.

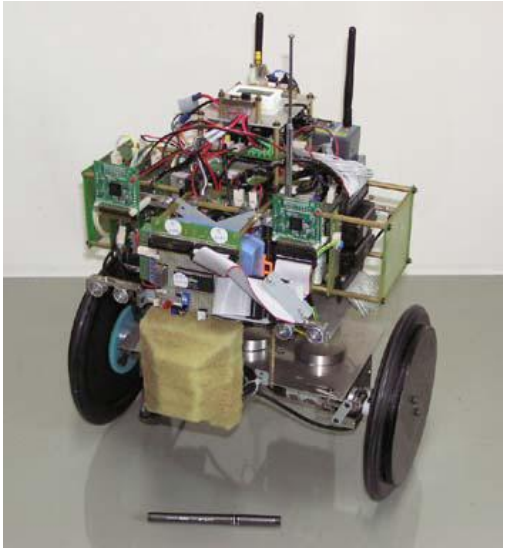

Figure 2.2 – Two Wheel Transporter in Ching Yun University, [7]

Figure 2.3 - Wheeled Inverted Pendulum, [12]

As a transporter system, it is driven semi-autonomously by the driver on it. The driver determines the speed and direction of movement of the vehicle by leaning forward and backward. Most of the transporters are combined of standing base and handlebar which make the driver feel comfortable [23-27]. Also, steering mechanism is generally mounted on the handlebar. However, the vehicle in [28] only consists of a standing base. Steering is provided by shifting center of gravity (COG) of driver. The study [13] discusses the system both as a transporter and as a robotic platform which carries goods.

Some studies which are inspired by Segway emphasize creating lightweight and low-cost systems. This is proven in [8, 13, 23, 28, 29, 30]. Their low-costs make them affordable and lightweight make them portable in anywhere.

Such two wheeled systems have a wide range of application area. It is clear that the transporter system is used for transportation. On the other hand, robotic system is used for telepresence applications by integrating camera as in [31]. Also, soccer player is made up in [17]. Some robotic systems are designed to carry load [11, 29]. They are used for educational purposes in some studies [10, 32]. Robotic systems generally turn into hybrid systems by combining with necessary components according to the application. Two wheeled robot with arms and waist is designed as a human assistant robot in [33]. The hybrid system in [34] is the robot combined with a manipulator. Two wheeled platform is also used for actuator of a humanoid robot [35]. Moreover, a system which is designed as both ground and aerial robot is studied in [36].

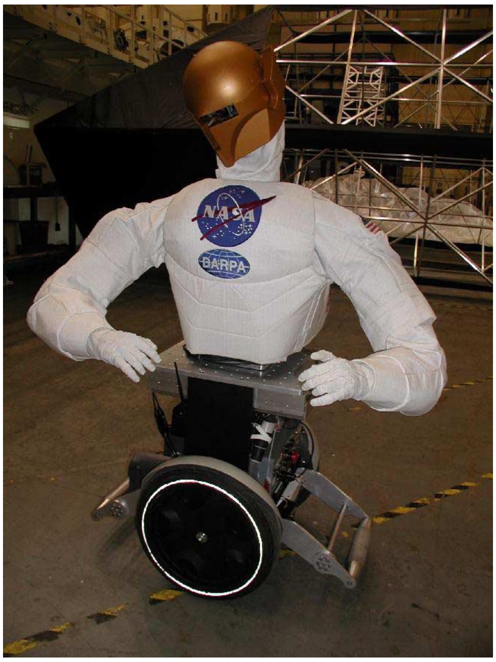

Figure 2.4 – Mobile Humanoid Robot Robonaut with Mobility Platform, [35]

Besides the two-wheeled system, similar studies about one-wheeled (unicycle) self balancing system [37] and balancing on ball systems [38, 39] exist in the literature. As a mechatronic system, it is interested by the academicians. Its stability problem and the interaction among controller algorithm, hardware and software make it popular to study on.

Mathematical model of the system is derived in order to design a controller. Many studies apply Lagrange equations while deriving mathematical model [4, 9, 20, 29, 37, 39, 40, 41]. On the other hand, mathematical model is derived from Newton's law of motion in some studies such as [15, 21, 32, 42]. System's states are determined as linear displacement and linear velocity in longitudinal direction, angle and angular rate related to pitch dynamics [3, 15]. Also, yaw angle and yaw angular rate are considered in some studies [1, 14, 23, 30, 39, 43]. Mathematical model is the representation of the real system. Therefore, system's parameters such as inertia are important in order to make model more accurate. Inertia of the system is determined by calculating as in [26, 27] or testing as in [20, 39].

Designing controller is the crucial part of the system. The main problem stability is satisfied by the controller. Although this system is highly non-linear, linear controllers are generally applied to the system after linearization because of its low-cost and less complexity. However, non-linear controllers are also attempted in [30]. Most of the studies focus on auto-balancing control. Besides auto-balancing control, controllers are used for tracking control in some studies as [24, 29, 44]. Tracking the reference input is achieved here. Yaw dynamics are considered for trajectory tracking control in [14]. Also, studies about steering control related to yaw motion exist in [25, 28, 29, 30]. Many kinds of linear control algorithms are studied on this system. One of the most common controllers is PID type algorithm as in [6, 13, 21, 27, 29, 31, 39 and 45]. This algorithm is easily implemented on the system. Moreover, PD controller is preferred in [10, 32, 46]. The reason of not using Integral parameter "I" is stated in [46] as its demand of large amount of processing power. Other common controller algorithm is LQR which depends on state feedback controller approach. It is designed and implemented in [4, 5, 11, 20, 16, 33, 37, 44, 47, 48]. State feedback controller makes the system robust. Observer is used in order to estimate the states in [4, 24]. Also, it is used as disturbance estimator in [12] and [34]. In addition, some studies use both LQR and PID algorithms to compare their performance such as in [42] and [49]. Pole placement method is used in [1] and [15]. H_2 and H_{inf} methods are used in [18] and [19], respectively. Other controller methods implemented on the system are fuzzy control as in [7, 8, 24] and adaptive control in [14, 23, 30].

The essential aim is to stabilize pitch angle in the system. Thus, necessary data must be taken from sensors. The main sensors of the system are accelerometer and gyroscopes which measure the angle and angular rate of the body, respectively. Most of the studies, [12, 13, 28] use both of these sensors together. However, accelerometer gives noisy data and gyroscope causes drift. Thus, these two sensors are combined with a complementary filter in order to get more accurate data in [12]. Kalman filter is used for sensor fusion during combination of gyroscope and inclinometer [2]. Also, advanced sensor units as inertial measurement units comprising both gyroscope and accelerometer are used in [15] and [39]. These units give filtered data. The studies which only use accelerometer or gyroscope also exist. Gyroscope is used alone [33, 42, 48] while accelerometer is used in [4, 26, 29]. There are also different sensors to measure tilt angle instead of accelerometer in the literature. Inclinometer detects the pitch angle in [2, 7, 9, 18]. Also, tilt angle is obtained from infrared range sensors in [6] and [45].

In addition to the above sensors, many different types of sensors are used in the system. Encoders measure the linear displacement of the system and linear velocity is also obtained by encoders [4, 9, 10, 12, 20, 42, 43, 44, 48]. Also, Hall Effect sensor is used instead of encoder [25]. Sensors show variety with respect to purpose of the system. Potentiometer coupled to handle bar detects the yaw rotation reference [32]. Camera [17, 21, 22, 31] and laser range sensor [43] detect the environment of the robot to navigate. Moreover, bluetooth [9, 20] and xBee [46] modules implement wireless communication.

All processing are carried out by the embedded controller hardware. Microcontrollers are generally preferred in literature because of affordability [5, 7, 8, 9, 10, 27, 28, 38]. Microcontrollers manufactured by Microchip and Atmel are used in many researches [4, 26, 29, 46]. Digital Signal Processing (DSP) board is used for real time applications as in [14, 15, 18, 32]. Besides DSP, field programmable gate arrays (FPGA) is used as the controller hardware of the systems in [1] and [15]. In addition, single board computers in PC104 form are employed for real time control with

Matlab code and used in [13, 15, 36, 44]. Also, dSpace board is used in literature [47].

Most preferred software implemented the system is Matlab. Simulations are performed in Matlab/Simulink [4, 9, 11, 44, 47]. Also, Matlab/SimMechanics is used for dynamic modeling [4]. Controller gains are determined in Matlab environment [16, 50]. Moreover, Real-Time Windows Target (RTWT) is the platform for real time applications [4, 15, 44]. Besides Matlab, control programs are written in C and assembly languages in [34] and [3], respectively.

2TEA is different from most of the studies in the literature due to its design to operate both manned and unmanned. In addition, this operation is expected to be satisfied by state feedback control and observer design.

CHAPTER 3

PHYSICAL SYSTEM

2TEA is a mechatronic system which is discussed as mechanical and electronic parts. It is similar to a scooter vehicle as its structure. Also, it consists of various kind of electronic hardware. Physical system of 2TEA includes structure, motors, motor controllers, batteries, sensors, controller hardware and software.

2TEA is combined of two parts. Motors, motor controllers, batteries, PC/104 single board computer and inertial measurement unit are placed in the bottom part where the driver stands. Two holders create the top part of the system. Also, push buttons for steering are on the top part. The system is actuated by two 400 watt, 24V geared dc motors.

The controller hardware is a single board computer with PC104 architecture which is commonly used in the military aerial and land vehicles. Data acquisition is satisfied by this computer. Also, it is compatible with Matlab real time platform xPC Target. The code generated by this software in the host computer is transferred to the target computer placed in the system, and the system is run by these commands. Communication between target computer and host computer satisfies via serial port. xPC Target is preferable to other real time application platform Real-Time Workshop because of the ability to disconnect from host computer.

An inertial measurement unit that combines three-axial accelerometer, three-axial gyroscope and three-axial magnetometer is used as sensor of the system. The required power is satisfied by Li-Po batteries. Motors and motor controllers are fed by seven 3.7V battery packs while three 3.7V batteries feed computer and sensor.

System construction is demonstrated in Figure 3.1. The sensor detects the system states and sends this information to the single board computer where the real time control algorithm runs. After data processing, the motor controller is activated to generate signal which actuates the motors. Actuated motors rotate the wheels and system moves.

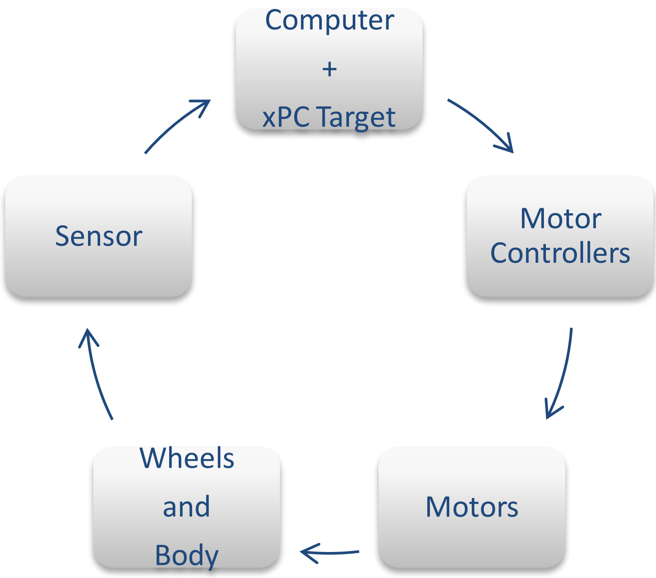

Figure 3.1 - System Construction

Detailed descriptions about physical system are explained in this chapter.

3.1 Mechanical Structure

Mechanical structure of 2TEA was constructed as a term project in a technical elective course by undergraduate students. Structure of 2TEA is like an inverted pendulum. It stands on two wheels which place right and left sides. Figure 3.2 demonstrates the system. Top part of the body is created by holders where push buttons are mounted for steering. Driver stands on the bottom part whose shape looks like a box. All hardware is placed in the bottom part. Also, a lifting mechanism is placed in front of 2TEA. It is explained in this chapter later, in detailed.

Chassis is made of sheet metal and aluminum. Aluminum is used in the frame of bottom part, and sides are covered by sheet metal plate. Total height of body is approximately 1.3 m from ground. Also, total mass of 2TEA is 36.11 kg. Inertia of system was determined by inertia test which is mentioned in chapter 4, in detailed. Although system is able to stabilize on two wheels, casters were used for safety during experiments.

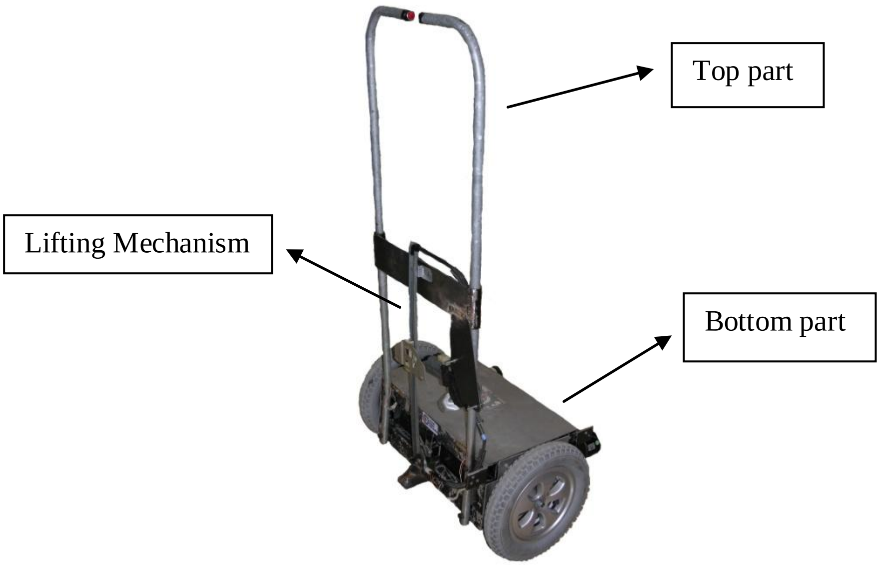

Figure 3.2 - 2TEA designed by Department of Mechatronics Engineering in Atılım University

3.2 Sensor

System responses are detected by sensors. The main problem of such a system is stabilizing pitch angle, so a sensor that measure this response is essential. Detecting angular rate is also necessary for stabilization.

Inertial Measurement Unit (IMU) is used to measure Euler angles and angular rate in 2TEA. Microstrain 3DM-GX1 is employed as IMU. It combines three angular rate gyros with three orthogonal DC accelerometers, three orthogonal magnetometers, multiplexer, 16 bit A/D converter, and embedded microcontroller, to output its orientation in dynamic and static environments. The embedded microcontroller filters the outputs. Also, it has ability to compensate temperature. Figure 3.3 demonstrates IMU.

IMU connects to computer via RS-232. Sampling rate of sensor is 100 Hz in this system. IMU gives pitch angle as radian and angular rate about pitch axis as radian/second.

Figure 3.3 - Microstrain 3DM-GX1 IMU

3.3 Encoder

Velocity is one of the states of 2TEA and velocity data is necessary in order to design a full-state observer. Thus, US Digital optical kit encoder is used to acquire this data in the system. Encoder measures displacement and velocity is derived from displacement. It is a 2 channel quadrature incremental encoder whose resolution is 100 count-per-revolutions. It is mounted on the right motor shaft and the motor has a gear box with 1:28.7 gear ratios. Hence, 4x100x28.7 peaks correspond to one wheel rotation of 2TEA. Following equation demonstrates the conversion between encoder output and wheel displacement.

$$Displacement = \frac{2\pi r}{11480} \tag{2.1}$$

3.4 DC Motor and Motor Driver

2TEA is actuated by two 400 watt, 24 volts brushed DC motors with gear assembly. It has 1:28.7 gear ratios which provide high torque. This motor is used in electrical wheelchairs. Motor is shown in Figure 3.4.

Maxon Motor 4-Q-DC Servoamplifier ADS 50/10 shown in Figure 3.5 is a powerful servoamplifier for driving permanent magnet DC motors from 80watts up to 500 watts. It supplies maximum 20A, continuous 10A current. Efficiency of up to 95% is achieved thanks to MOSFET power transistors incorporated in the servoamplifier. It has high PWM frequency of 50 kHz.

Figure 3.4 – Faz Elektrik DC Motor

Maxon Motor Controller provides amount of current driven by motor. Also, this motor controller offers four operation modes. IxR compensated speed control is chosen for 2TEA. IR compensation is positive feedback that rise control output voltage with increasing output current. This means motor speed is stable from no-load to full load conditions.

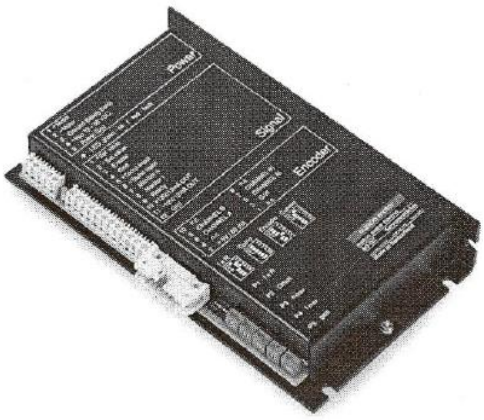

Figure 3.5 - Maxon Motor Controller

3.5 Controller Hardware

Diamond System Prometheus single board computer with PC/104 architecture is used as controller hardware. Single board computer means CPU and data acquisition are on a single board. This provides advantage in size, weight, power consumption and cost. Also, this compact form computer is reliable and rugged in many applications. ZFx86 processor runs at 100 MHz. Data acquisition is satisfied by 16 single-ended / 8 differential 16-bit analog inputs, 4 12-bit analog outputs, and 24 programmable digital inputs/outputs. Also, Prometheus includes 4 serial ports and 2 usb ports. Thus, it provides sufficient inputs/outputs to collect data and to control system.

Figure 3.6 – Prometheus Single Board Computer

Prometheus is compatible with Matlab xPC Target platform. It is used as a target computer in 2TEA. Controller algorithms are sent from desktop computer acting as a host computer to Prometheus. Also, Prometheus is covered by an enclosure Pandora.

3.5.1 Quadrature Encoder Input PC/104 Data Module

Encoder data is acquired by Real Time Devices DM6814 board. It is a PC/104 data module which can be mounted on Prometheus. It has the ability to count pulses of 3 16-bit incremental encoders. It is compatible with Matlab software.

3.6 Controller Software

Matlab/Simulink and xPC Target platform are the software to generate code in order to control system. The xPC Target platform implements real time applications. Target computer does not require any operating system. Instead, the xPC Target kernel which provides real time operating system runs in the target computer. Parameters are calculated in Matlab. Then, controller is designed in a Simulink model. After that, model is compiled and generated executable code is downloaded from host computer to the target computer. After all, the application runs in real time. Also, this environment allows tuning parameters while the system is running.

3.7 Power Unit

Li-Po batteries generate required power for actuators and other components. It is preferred due to its light weight and small thickness. Seven 3.7 V, 11 Ah Li-Po battery cells are packed in order to acquire 25.9 volts to feed motors and motor controllers. Also, three cells 1.6 Ah, 11.1 volts battery feeds sensor and single board computer.

Indeed, single board computer and sensor run at 5 volts, so a power card providing 5 volts output is required. Real Time Device power supply module is used to get 5 volts.

3.8 Lifting Mechanism

Lifting system which is placed in front of 2TEA is thought to carry goods. It is a window lifter with high torque, 12 volts brushed dc motor. Pololu High Power Motor Driver 18v15 drives this motor. Driver can run the mechanism up and down while carrying load.

CHAPTER 4

MATHEMATICAL MODELLING

Mathematical model is the representation of real systems. It provides information about the characteristics of the system and describes dynamical behavior of system. It denotes the system's states, inputs and outputs mathematically. Mathematical model is used to describe, analyze and simulate the system. Control systems are designed based on the mathematical model and simulations.

4.1 Mathematical Model of 2TEA

Longitudinal and pitch dynamics of 2TEA are modeled in order to stabilize and control them. Positive directions of motion are depicted in Figure 4.1. Mathematical model of 2TEA is derived by applying Newton's Law of Motion. The sum of all external forces and moments are the resultant (total) force and resultant moment with respect to Newtonian dynamics.

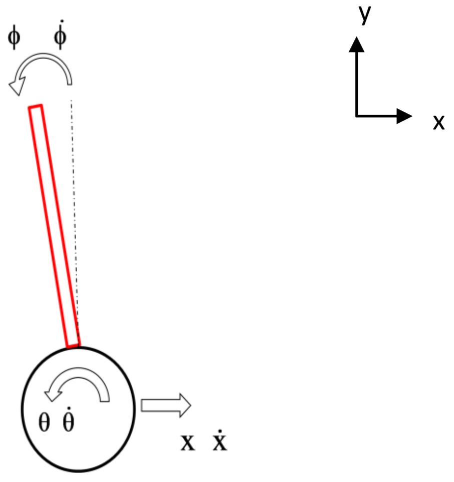

Figure 4.1 - Positive Directions of Motion

2TEA is a rigid body which consists of wheel and pendulum. Equations of motion of both are derived to model the whole system. Free body diagram (FBD) of wheel shows all forces and torques on the wheel, in Figure 4.2.

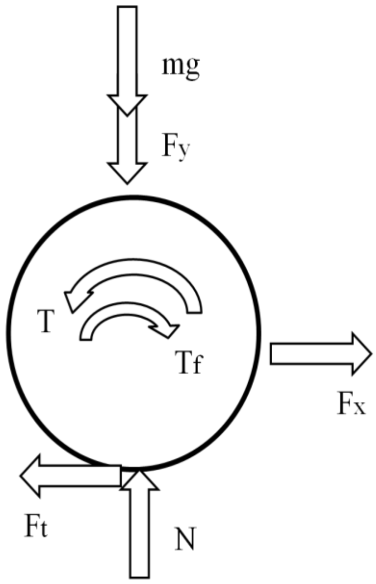

Figure 4.2 - FBD of Wheel

Resultant force in x-direction equals to inertia in positive direction with respect to Newton's law.

$$\sum F_x = ma \tag{4.1}$$

$$m\ddot{x} = F_x - F_t \tag{4.2}$$

There is no motion in vertical direction. Thus, sum of forces in y-direction equals 0.

$$\sum F_y = ma \tag{4.3}$$

$$0 = N - F_y - mg \tag{4.4}$$

F_x and F_y are the forces on the center of wheel, and F_t is the traction force occurred between wheel and surface. In addition to translational motion, rotational motion is considered.

$$\sum M = I\alpha \tag{4.5}$$

$$I_w\ddot{\theta} = T - \ rF_t - T_f \tag{4.6}$$

In the above equation T_f denotes the frictional torque on the rotation axis.

$$T_f = b(\dot{\theta} - \dot{\phi}) \tag{4.7}$$

Equations of motion of the body are derived by applying same rules. Resultant forces in x and y-directions, and resultant moment are as below. Figure 4.3 is FBD of body which acts like a pendulum.

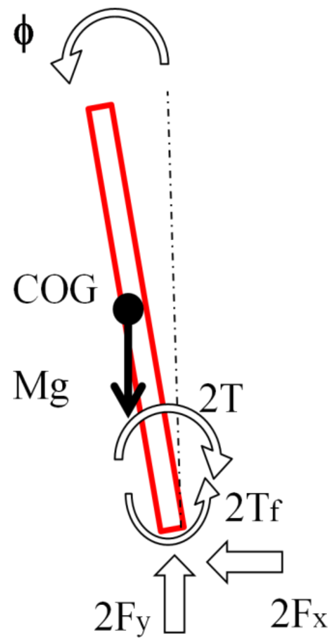

Figure 4.3 - FBD of body

$$\sum F_x = ma \tag{4.8}$$

$$M\ddot{x}_b = -2F_x \tag{4.9}$$

$$\sum F_y = ma \tag{4.10}$$

$$M\ddot{y}_b = 2F_y \ - \ Mg \tag{4.11}$$

$$\sum M = I\alpha \tag{4.12}$$

$$I_b\ddot{\phi} = -2T - 2F_xLcos(\phi) + 2F_yLsin(\phi) + 2b(\dot{\theta} - \dot{\phi}) \tag{4.13}$$

Kinematic equations related to wheel and body motions are below;

$$x_b = -Lsin(\phi) \tag{4.14}$$

$$\dot{x}_b = \dot{x} - \dot{\phi}Lcos(\phi) \tag{4.15}$$

$$\ddot{x}_b = \ddot{x} - \ddot{\phi}Lcos(\phi) + \dot{\phi}^2Lsin(\phi) \tag{4.16}$$

$$y_b = Lcos(\phi) \tag{4.17}$$

$$\dot{y}_b = -\dot{\phi}Lsin(\phi) \tag{4.18}$$

$$\ddot{y}_b = -\ddot{\phi}Lsin(\phi) - \dot{\phi}^2Lcos(\phi) \tag{4.19}$$

Forward and pitch dynamics are considered in the model. It is assumed that there is no slippage between wheel and ground. The relation between linear and angular motion is defined as;

$$x = r\theta \tag{4.20}$$

$$\dot{x} = r\dot{\theta} \tag{4.21}$$

$$\ddot{x} = r\ddot{\theta} \tag{4.22}$$

Nonlinear equations of motion of 2TEA are obtained from the above equations.

$$I_w\left(\frac{\ddot{x}}{r}\right) + r\{-0.5M[\ddot{x} - \ddot{\phi}Lcos(\phi) + \dot{\phi}^2Lsin(\phi)] - m\ddot{x}\} + b(\dot{\theta} - \dot{\phi}) = T \tag{4.23}$$

$$I_b\ddot{\phi} + 2\{-0.5M[\ddot{x} - \ddot{\phi}Lcos(\phi) + \dot{\phi}^2Lsin(\phi)]\}Lcos(\phi) - 2\{0.5M[-\ddot{\phi}Lsin(\phi) - \dot{\phi}^2Lcos(\phi)] + 0.5Mg\}Lsin(\phi) - 2b(\dot{\theta} - \dot{\phi}) = -2T \tag{4.24}$$

The torque, T applied to 2TEA is generated by a dc motor. However, the input is considered as voltage in this system. Therefore, a function such as T = f(V) is required. DC motor model is derived in order to define torque in terms of voltage. DC motor model is depicted in Figure 4.4.

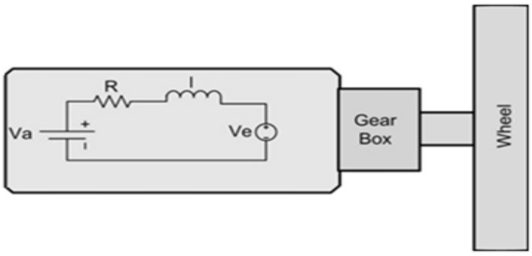

Figure 4.4 - DC Motor Model, [36]

Electrical characteristic of DC motor is used for modeling. T=f(V) is obtained by the relations between torque and armature current; and voltage and armature current.

$$T_m = K_t i \quad (4.25)$$

$$V - Ri - l\frac{di}{dt} - V_e = 0 \quad (4.26)$$

Equation 4.26 is based on Kirchhoff's Law. Also, back EMF voltage, V_e is related to armature velocity.

$$V_e = K_e \dot{\theta}_m \quad (4.27)$$

It is assumed that the dc motor has a small *l* value. After rearranging Equation 4.26, current is stated as below;

$$i = \frac{V}{R} - \frac{K_e}{R}\dot{\theta}_m \quad (4.28)$$

Substituting Equation 4.28 into Equation 4.25 gives following equation;

$$T_m = K_t\left(\frac{V}{R} - \frac{K_e}{R}\dot{\theta}_m\right) \quad (4.29)$$

θ_m denotes rotation of motor shaft. Also, it can be written in terms of rotation of wheel and pitch angle of body as following;

$$\theta_m = n\theta - \phi \quad (4.30)$$

$$\dot{\theta}_m = n\dot{\theta} - \dot{\phi} \tag{4.31}$$

The torque mentioned in the above equations is related to motor shaft. However, torque applied to the wheel is considered in the mathematical model of 2TEA. Thus,

$$T = nT_m \tag{4.32}$$

$$T = \frac{nK_tV}{R} - \frac{nK_tK_e}{R}(n\dot{\theta} - \dot{\phi}) \tag{4.33}$$

After substituting Equation 4.33 into Equation 4.23 and 4.24, equations of motion of 2TEA related with voltage are obtained.

$$I_w\left(\frac{\ddot{x}}{r}\right) + r\{-0.5M[\ddot{x} - \ddot{\phi}Lcos(\phi) + \dot{\phi}^2Lsin(\phi)] - m\ddot{x}\} + b(\dot{\theta} - \dot{\phi}) - \frac{nK_tV}{R} + \frac{nK_tK_e}{R}(n\dot{\theta} - \dot{\phi}) = 0 \tag{4.34}$$

$$I_b\ddot{\phi} + 2\{-0.5M[\ddot{x} - \ddot{\phi}Lcos(\phi) + \dot{\phi}^2Lsin(\phi)]\}Lcos(\phi) - 2\{0.5M[-\ddot{\phi}Lsin(\phi) - \dot{\phi}^2Lcos(\phi)] + 0.5Mg\}Lsin(\phi) - 2b(\dot{\theta} - \dot{\phi}) + 2\frac{nK_tV}{R} - 2\frac{nK_tK_e}{R}(n\dot{\theta} - \dot{\phi}) = 0 \tag{4.35}$$

Mathematical modeling is required to design a controller. Linearization of nonlinear state equations is performed to obtain a linear state space model. Nonlinear dynamic equations are rearranged as follows.

$$\ddot{\phi} = \frac{1}{\beta}\left\{MLcos(\phi)\ddot{x} + \left(\frac{2b}{r} + \frac{2n^2K_tK_e}{rR}\right)\dot{x} - \left(\frac{2bR + 2nK_tK_e}{R}\right)\dot{\phi} + MgLsin(\phi) - \frac{2nK_tV}{R}\right\} \tag{4.36}$$

$$\beta = I_b + ML^2 \tag{4.37}$$

$$\ddot{x} = \frac{1}{\alpha}\left\{-0.5MrLcos(\phi)\ddot{\phi} + 0.5MrLsin(\phi)\dot{\phi}^2 - \left(\frac{bR + n^2K_tK_e}{rR}\right)\dot{x} + \left(\frac{bR + nK_tK_e}{R}\right)\dot{\phi} + \frac{nK_tV}{R}\right\} \tag{4.38}$$

$$\alpha = \frac{I_w}{r} - 0.5Mr - mr \tag{4.39}$$

States of 2TEA are determined as linear displacement, linear velocity, pitch angle and pitch angular rate. Voltage is the input of system. It is assumed that the left and right motors are actuated by the same voltage input. The system is defined as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{4.40}$$

2TEA is linearized around zero tilt angle and angular rate by using Taylor Series Expansion method. After linearization, state matrix A and input matrix B are obtained. This linearization method is implemented on Matlab. State space representation of linearized system is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & A2,2 & A2,3 & A2,4 \\ 0 & 0 & 0 & 1 \\ 0 & A4,2 & A4,3 & A4,4 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ B2,1 \\ 0 \\ B4,1 \end{bmatrix} V \tag{4.41}$$

The output matrix C depends on the sensors on the system. Angular velocity and angular displacement are measured by IMU. Also, linear velocity and linear displacement can be measured by encoder. During experiments, IMU is utilized generally. In addition, GPS module is also available and it will be placed on the system in future studies. Thus, four outputs can be shown in state space model.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \tag{4.42}$$

4.2 System Parameters

System parameters are important in order to make system model more accurate. Some of the parameters in equations of motion are known or can be found easily. Mass of body and wheel are measured easily, for instance. On the other hand, inertia of body and wheel, and motor parameters are found experimentally.

4.2.1 Motor Parameters:

Electrical parameters of motor are not expressed in datasheet. It is assumed that right and left motor have same characteristics, so experiments were only performed on the left motor.

Resistance of motor was measured via a multimeter. It was determined as 1.4Ω, approximately.

An encoder was built in order to determine back EMF constant of motor. Encoder code wheel was mounted on the wheel, and Hamamatsu photo reflector was used to detect pulses. Code wheel with four ticks was enough to be detected by photo reflector. Construction of encoder is seen in Figure 4.5.

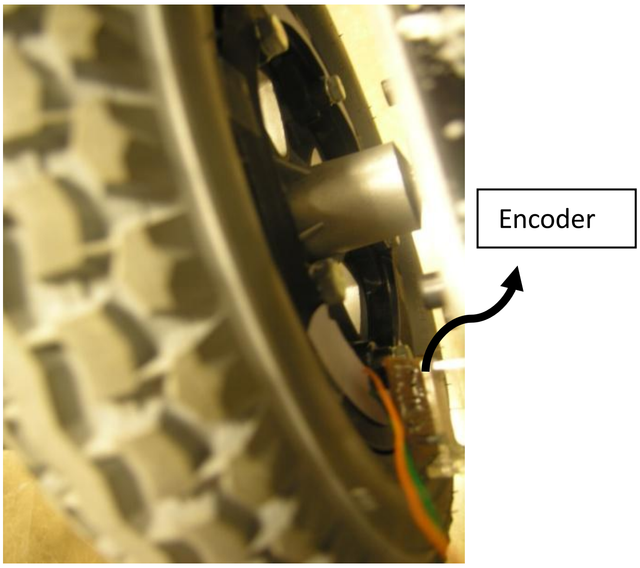

Figure 4.5 - Encoder

The period of pulses were observed as 240 msec. at the maximum speed via oscilloscope. This means 480 msec. for 1 revolution. It gave 125 rpm or 13.09 rad/sec. for the output shaft. Figure 4.6 shows experimental setup.

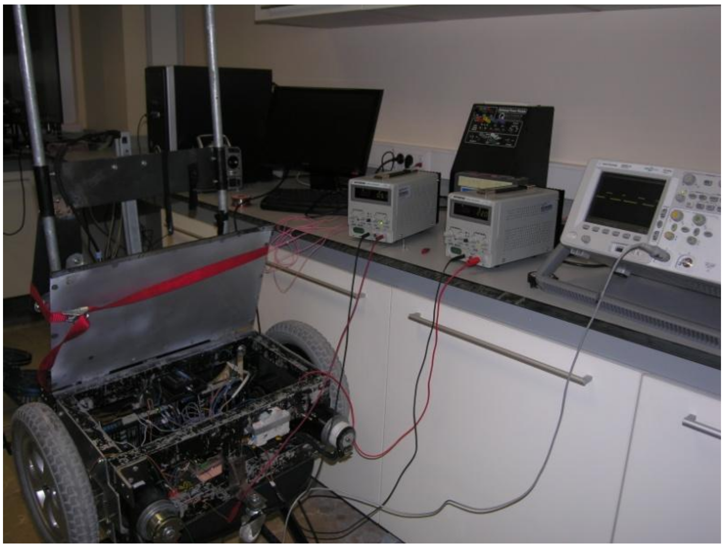

Figure 4.6 - Back EMF Constant Experimental Setup

The formula related to back EMF constant is

$$V - Ri - L\frac{di}{dt} - V_e = 0 \tag{4.43}$$

$$V_e = K_e\dot{\theta}_m \tag{4.44}$$

Input voltage was measured as 19.8 V and motor armature current was obtained as 0.6865 A. The equations are rearranged as below;

$$V - Ri = K_e\dot{\theta}_m \tag{4.45}$$

$$19.8 - 1.4 * 0.6865 = K_e * 125 * 28.7 \tag{4.46}$$

Hence K_e is found as 0.05 V/rad/sec. The parameters are related to motor shaft, so velocity of output shaft was converted to velocity of motor shaft by multiplying gear ratio. It is assumed that motor torque constant, K_t has same magnitude with K_e.

4.2.2 Inertia Tests:

Moments of inertia cannot be determined easily. A test system is required to construct for that. The applied technique is known as bifiliar pendulum. Moment of inertia of the body about pitch axis of rotation should be calculated.

2TEA was hung to make it free to oscillate on pitch axis. Two ropes were used to hang it upward, and they were tied from the points which were approximately same distance to rotation axis of 2TEA. Figure 4.7 shows the experimental setup for body.

After hanging, body was released from a small initial angle to oscillate. The response of the system was collected by IMU. Period of the oscillation is derived from the collected data in Figure 4.8.

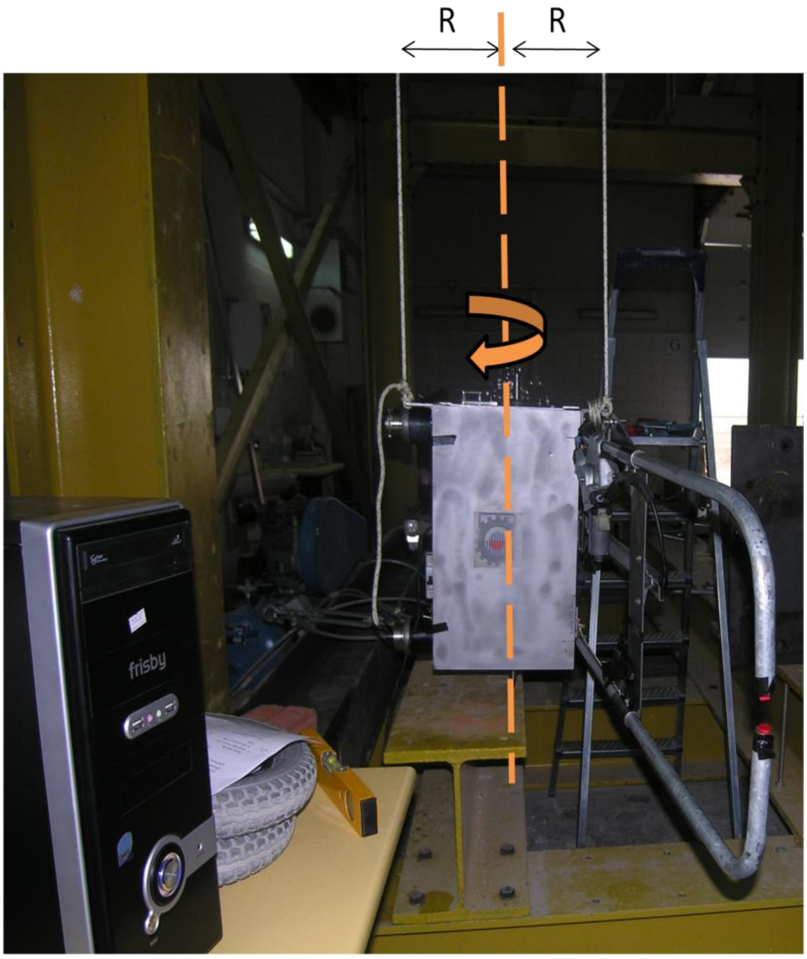

Figure 4.7 – Inertia Test Setup for Body

Period of this oscillation is found as 3.13 s. Inertia is related to length of rope L_i, mass of the system m_i, hanging distance from rotation axis R_i, gravity g, natural frequency $2\frac{\pi}{T_n}$. Then, the formula is following;

$$J = \left[\frac{T_n}{2\pi}\right]^2 \frac{m_i g R_i^2}{L_i} \tag{4.47}$$

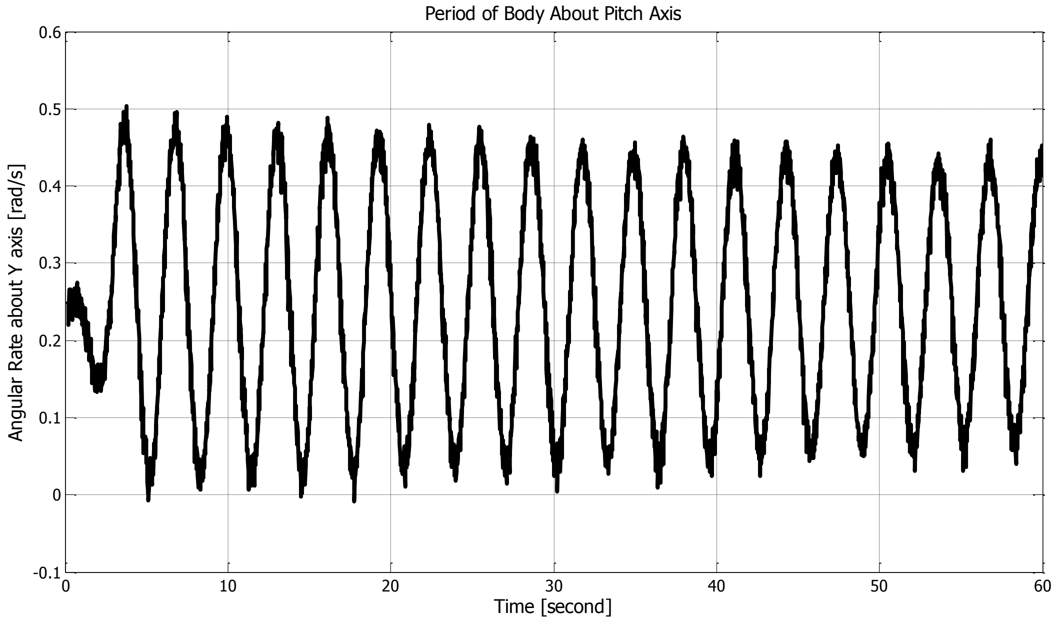

Figure 4.8 - Period of Body about Pitch Axis

Table 1 - Parameters for body inertia test

Length L_i:	1.37 m
Radius R_i:	0.2 m
Mass m_i:	31.95 kg
Gravity g:	9.8 m/s^2
Period T_n:	3.13 s

After applying Equation (4.47), moment of inertia of body is found as 2.2686 kg.m^2.

Similar experiment was performed to determine moment of inertia of wheel. Figure 4.9 shows inertia test setup for wheel.

Figure 4.9 - Inertia Test Setup for Wheel

Data about wheel during inertia test is seen in Figure 4.10. Period of oscillation is determined as 1.45 s.

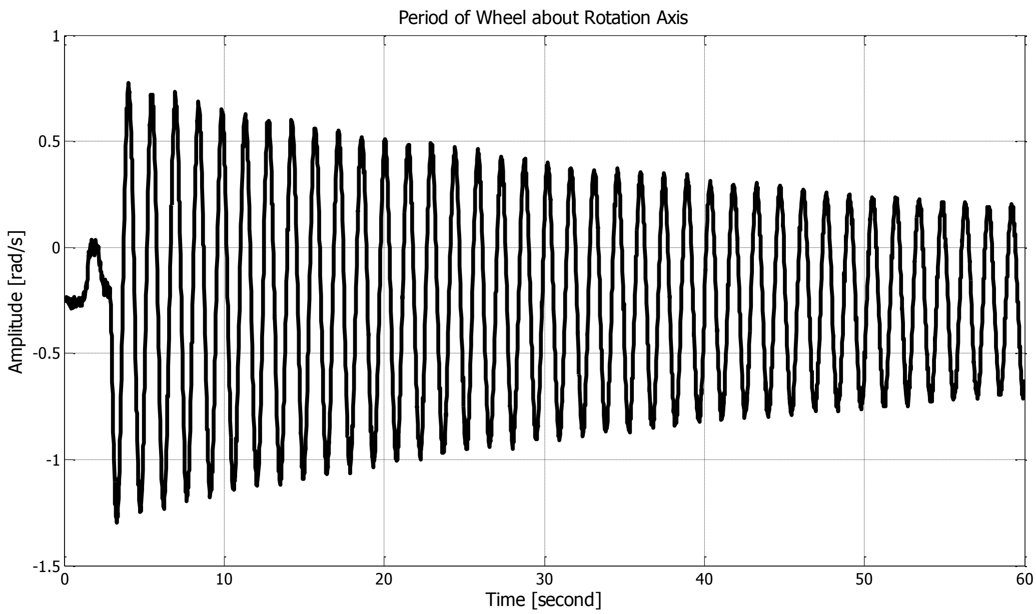

Figure 4.10 - Period of Wheel about Rotation Axis

Table 2 - Parameters for wheel inertia test

Length L_i:	1.67 m
Radius R_i:	0.16 m
Mass m_i:	2.08 kg
Gravity g:	9.8 m/s^2
Period T_n:	1.45 s

These parameters are used in Equation 4.47, and inertia of wheel is calculated as 0.0166 $kg.m^2$.

In addition, this experiment helped to find center of gravity of body. The body was hung on its equilibrium point and this was 0.1 m from wheel axis.

CHAPTER 5

CONTROLLER DESIGN AND SIMULATIONS

The characteristic of 2TEA can be examined with regard to mathematical model. System's pole location gives essential information about system to design a controller. System poles are acquired by eigenvalue of system matrix, A, or roots of characteristic equation. Matlab is a tool which is used to obtain them. Poles in below prove unstability of system because of having pole on the right-hand side of imaginary axis.

$$\text{Pole} = [0\ -3.6880\ 1.0758\ 3.6863] \tag{5.1}$$

2TEA is unstable, so both nonlinear and linearized model open-loop step responses go infinity. Response of pitch angle is seen in Figure 5.1. That result is obtained after modeling linearized and nonlinear system in Simulink, in Figure 5.2.

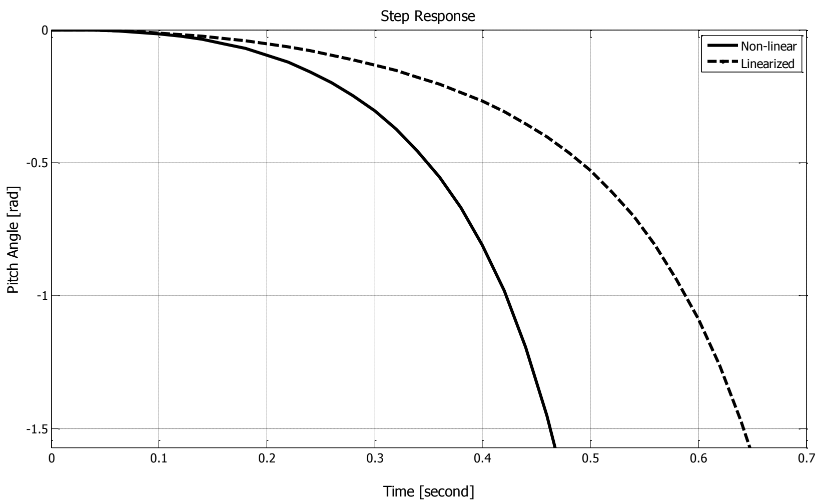

Figure 5.1 - Nonlinear and Linearized Open Loop System Response

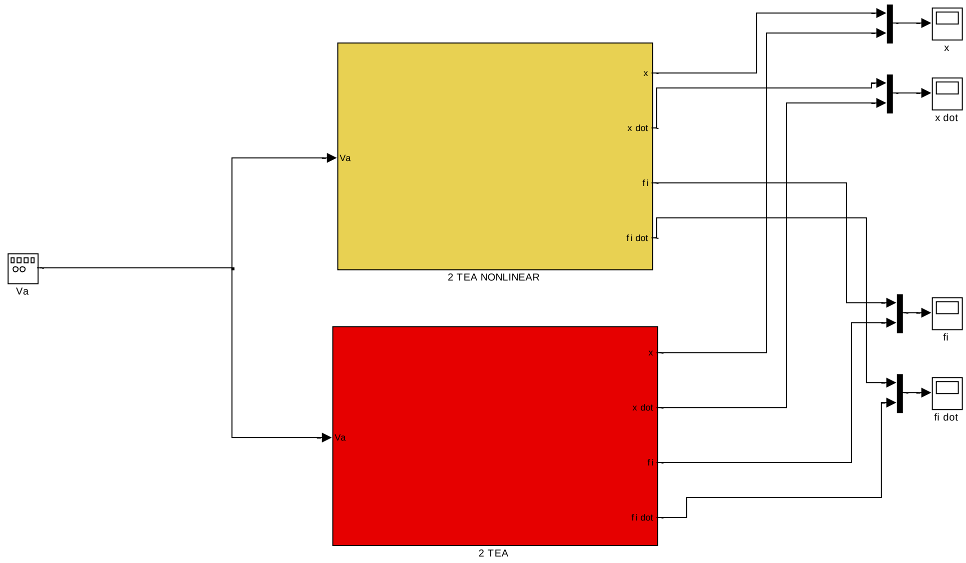

Figure 5.2 - Simulink Model of Nonlinear and Linearized Open-Loop Plant

Pitch angle and angular rate are considered as outputs in simulation. Thus, output matrix is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{5.2}$$

Controllability and observability analysis are important to design a controller [51]. Thus, controllability and observability of system must be analyzed. Matlab is utilized to perform analysis and design. Analysis exists in *Appendix A*. The rank of controllability and observability matrices determine the system's controllability and observability. They must equal to the number of states. 2TEA is controllable, but it is not completely observable in case of measuring only the pitch angle and pitch rate by IMU because of having less rank than number of states. Without using an encoder only IMU is placed on the system.

Considering the transporter mode of 2TEA, this is the manned operation mode, position x is not critical for the system. Rather translational velocity, pitch rate, and the pitch angle are critical for the stability and performance. First state variable is not coupled with the others in linearized state space model. Only the rate of first state variable is equal to the second state. Therefore, first state, x, is eliminated from the model and system is reduced to three states. In this case, the modified system is both observable and controllable. Then all poles can be moved to left-hand side of imaginary axis, and all states can be extracted from the observation.

Representation of reduced system is shown in Equation 5.3. Also, controllability and observability matrices of reduced system are described in Equation 5.4 and 5.5, respectively;

$$\begin{bmatrix} \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} A2,2 & A2,3 & A2,4 \\ 0 & 0 & 1 \\ A4,2 & A4,3 & A4,4 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} B2,1 \\ 0 \\ B4,1 \end{bmatrix} V \tag{5.3}$$

$$\text{Co} = [B \quad BA \quad BA^2] \tag{5.4}$$

$$\text{Ob} = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \tag{5.5}$$

State feedback controller with observer is designed on the Matlab Simulink. Observer is necessary to estimate unmeasurable states.

This reduced order system model is also appropriate for the first phase of the unmanned –robotic vehicle- operation. Reference velocity can be assigned and the system tracks that reference while maintaining the stability in pitch dynamics. Such an operation can be satisfactory if the system were used to work on a constraint path following a special line. Rotational commands are generated to keep the vehicle on path and meanwhile the vehicle tracks the velocity reference in a stable manner. This can be implemented in another study.

State feedback control method is directly related to system states. Control input is created by states. By this way, closed loop poles are moved to desired location. Also, state feedback control is employed to design an observer. The system is defined as in Equation 5.6. Moreover, input is described in Equation 5.7.

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \quad (5.6)$$

$$u = -Kx \quad (5.7)$$

Thus, desired closed loop system is described as below;

$$\dot{x} = (A - BK)x \quad (5.8)$$

First of all, desired poles of the system are determined. Dominant poles are chosen with respect to desired transient response of system. Response time, maximum overshoot, damping ratio (ζ) and natural frequency (w_n) play important role here. Formula of closed loop transfer function of dominant poles is following:

$$\frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \quad (5.9)$$

Dominant poles of 2TEA was determined as (-2 ± 4i). Unit step response of this is denoted in Figure 5.3.

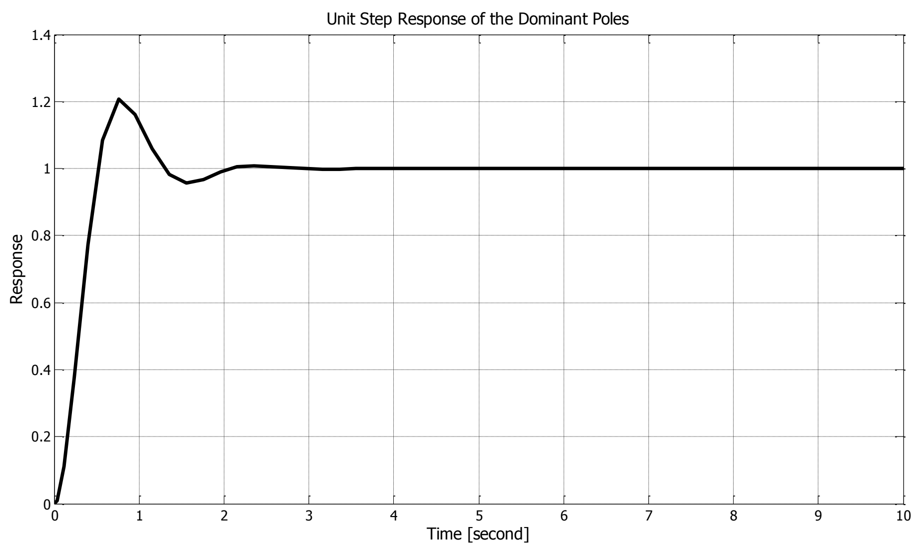

Figure 5.3 – Unit Step Response of Closed Loop Poles

It is seen from the Figure 5.3 that the response of these poles is fast enough for the system. Also, maximum overshoot is acceptable. Third pole is determined as far from dominant poles. Thus, it does not decrease the effects of dominant poles on the system. Then, desired poles were;

$$\text{Desired Poles} = [-2+4i \ -2-4i \ -10] \tag{5.10}$$

After that, state feedback controller gain is derived from these poles. Matlab is used to obtain controller gain K_r, in *Appendix A*.

$$K_r = [\ 109.5831 \ -346.3416 \ -163.9892] \tag{5.11}$$

In transporter mode, forward and backward motions, i.e. motion in x-direction are ignited by body lean. Pilot on the vehicle leans his/her body towards forward (backward) and the control system run the vehicle forward (backward) to maintain the stability. As the amount of lean increases forward velocity also tends to increase. Body lean in modeled as an additional external torque input about pitch axis in the state-space model. As the pilot starts leaning, the amount of external torque increases up to a constant value and as the pilot leans in opposite direction external torque decreases and as the pilot stands in an upright position on the vehicle the external lean torque is zero. A sample lean torque profile to drive 2TEA in +x direction and stop after a certain time is given in Figure 5.4.

At this point, considering the manned operation (i.e. transporter mode), it is assumed that the observer is not available to keep the controller simple and with low computational cost. Therefore, velocity feedback is not available. Then, K_r is modified in a following way.

$$K_{rm} = [-346.3416 \quad -163.9892] \tag{5.12}$$

Modified K_r is multiplied with modified state vector and fed back to system.

$$u = -K_{rm} \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} \tag{5.13}$$

Closed loop poles of this modified system are as follows;

$$\text{Modified Closed Loop Poles} = [5.2782 \quad -5.4157] \tag{5.14}$$

Simulink model of closed loop system and its response are shown in Figure 5.5 and Figure 5.6, respectively. Also, this system is a regulator system which has 0 reference input.

This simplified and low cost state feedback controller is also implemented on the physical system during the manned –transporter mode-. System response and details are revealed in the next chapter.

Feedback of reduced state vector with only pitch angle and pitch rate is not appropriate for unmanned –robotic vehicle- operations. Translational velocity should also be regulated for stable motion with desired performance. For unmanned operations additional sensors will be implemented on the system in the future studies. However, in the scope of this thesis, robotic operation with only an IMU in the sensor set is the starting point. It is assumed that during the unmanned operation, regulation of the pitch angle and angular rate are required for the desired performance. Regulation of the pitch angle at origin is not desired for example when the carrier handles load. This load generates an additional pitch moment which is considered as a disturbance. Estimation of such disturbances and disturbance rejection will be studied in another project. Therefore, it is out of scope of the thesis. In this study, unmanned 2 TEA is assumed to handle no disturbing loads. Therefore, corresponding controllers are designed according to this assumption and robustness is limited with also this assumption.

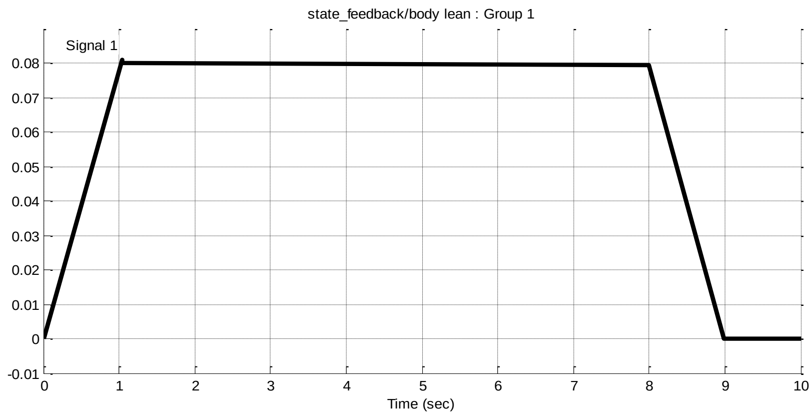

Figure 5.4 - Driver's Disturbance Simulation

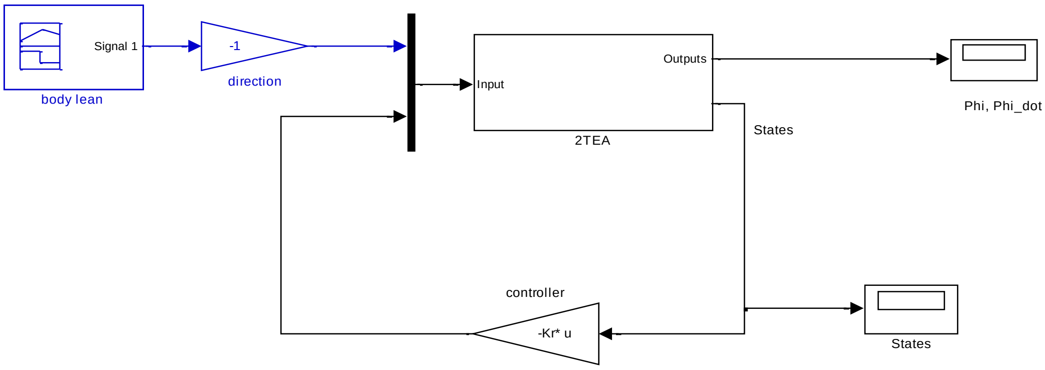

Figure 5.5 - Simulink Model of State Feedback Control

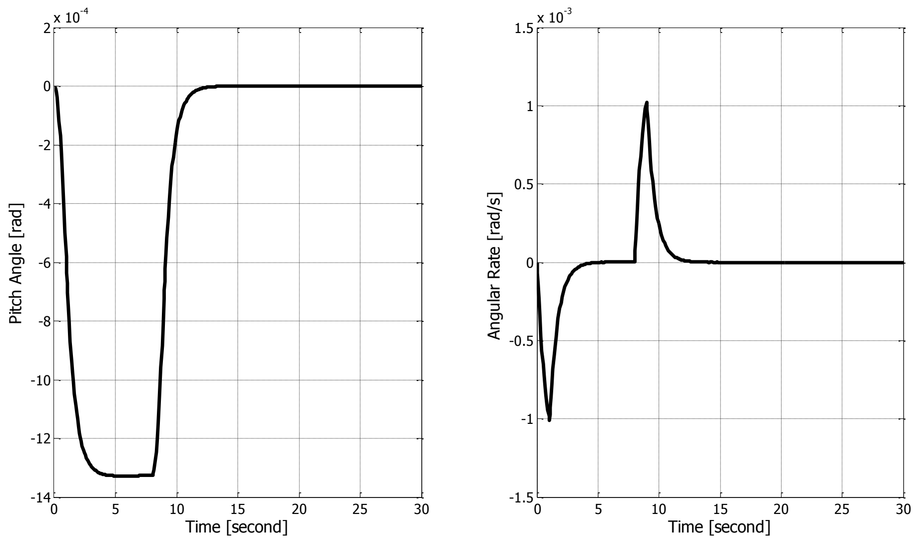

Figure 5.6 – Response of 2TEA

Velocity feedback is also essential for the unmanned robotic vehicle operation. Considering the reduced model with 3 states, the system is observable and controllable. An observer is designed to estimate state vector assuming that only pitch angle and pitch angular rate are measured. Observer poles are determined faster than the plant pole to make the estimation error converges to zero quickly and make the controller poles dominated [51]. Controlled system with observer generate control signal from estimated states ($\tilde{x}$). Control signal is given below;

$$u = -K\tilde{x} \tag{5.15}$$

System with observer is formed as following;

$$\dot{x} = Ax - BK\tilde{x} = (A - BK)x + BK(x - \tilde{x}) \tag{5.16}$$

Error between estimated and measured states and observer error equations are represented in Equation 5.17 and 5.18. K_e denotes observer gain here.

$$e = x - \tilde{x} \tag{5.17}$$

$$\dot{e} = (A - K_e C)e \tag{5.18}$$

After substituting Equation 5.17 into 5.16, state equation becomes;

$$\dot{x} = (A - BK)x + BKe \tag{5.19}$$

Combination of 5.18 and 5.19 gives observed state feedback control system, in 5.20.

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - K_e C \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{5.20}$$

In addition, observer is defined as;

$$\dot{\tilde{x}} = A\tilde{x} + Bu + K_e(y - C\tilde{x}) = (A - K_e C)\tilde{x} + Bu + K_e y \tag{5.21}$$

Unmanned system parameters such as inertia of body, mass of body are different than manned ones. Thus, new desired poles and controller gain are used in observed system. Observer poles of 2TEA were chosen two times further than desired closed loop poles. Also, initial condition is 0.01 rad.

Desired Poles for Unmanned Mode = [-1.5+2i -1.5-2i -2] (5.22)

Observer gain of 2TEA (K_{est}) was determined by Matlab, in *Appendix A*. Determining observer gain is similar to controller gain using the principle of duality.

Pitch angle and angular rate about pitch axis were observed to estimate all states. These states were fed back to the system. State feedback controlled system with observer was modeled and simulated in Simulink and Figure 5.7 depicts this model. Measured states are the input of observer system, control input which feeds the system plant is the output. Also, estimated states are gained from the observer plant.

Error between estimated states and measured states (pitch angle and angular rate) on simulation are shown in Figure 5.8. The error is very small and it goes zero. This means estimated states are reasonable and they are close to actual states. Moreover, outputs can be controlled as shown in Figure 5.9. Oscillation in outputs is damping and outputs reach zero as expected from regulator systems. Estimated velocity depicted in Figure 5.10 also makes sense.

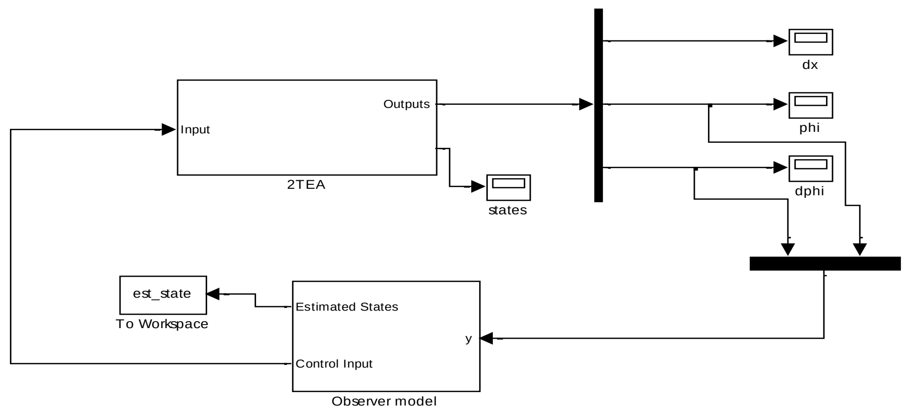

Figure 5.7 - State Feedback Controller with Observer

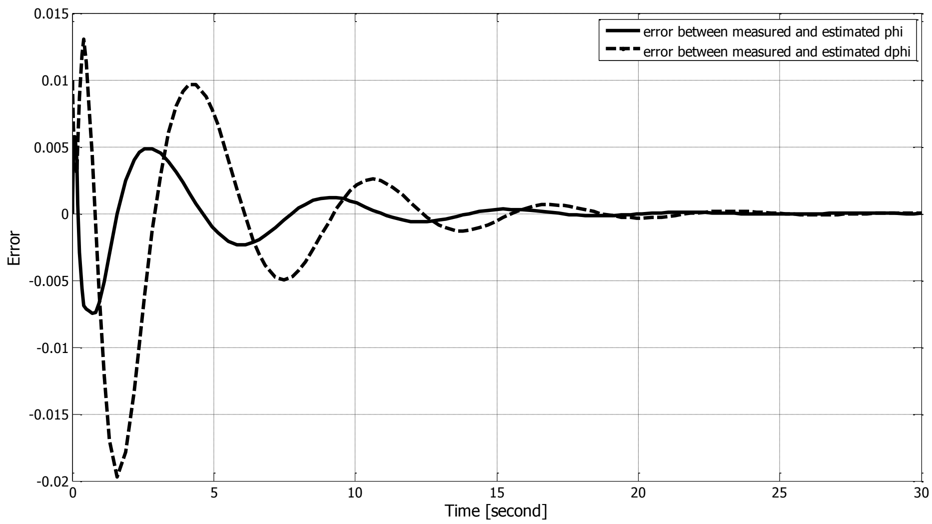

Figure 5.8 - Error between Measured and Estimated States on Simulation

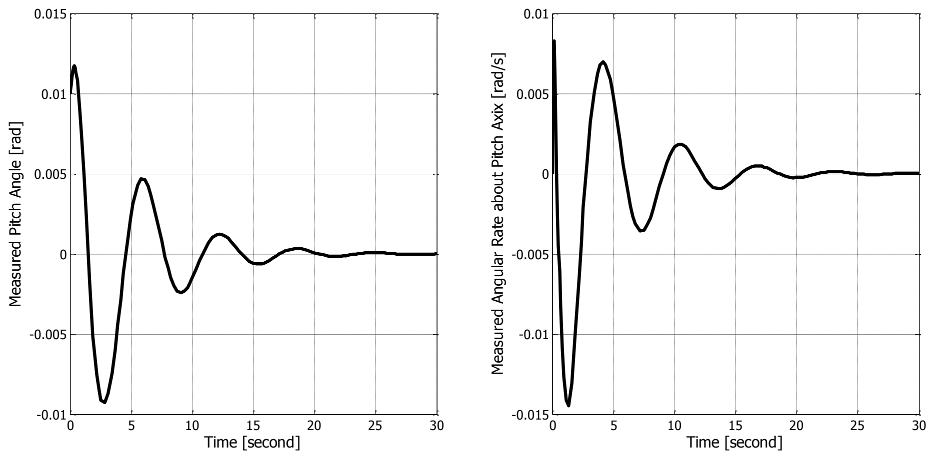

Figure 5.9 - Pitch Angle and Angular Rate about Pitch Axis

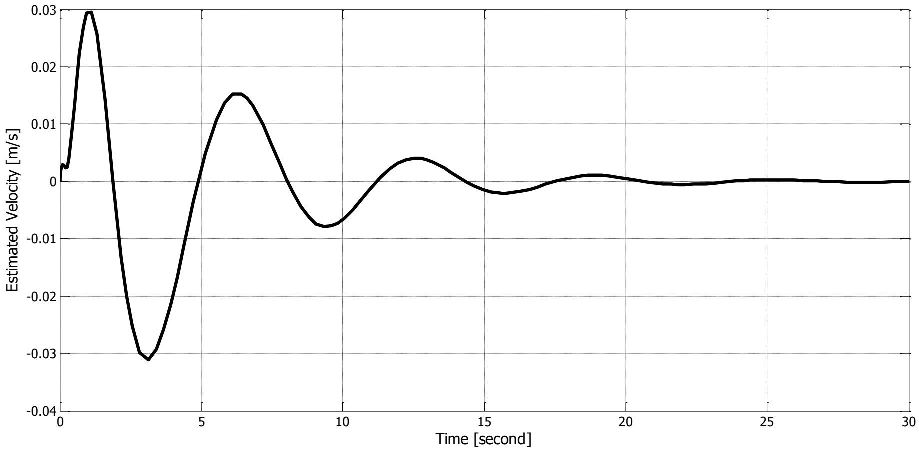

Figure 5.10 – Estimated Velocity

CHAPTER 6

EXPERIMENTS

Controller designs and simulations are implemented on the physical system. It is expected that the system move forward (or backward) when the driver leans forward (or backward) in transporter mode in order to stabilize body. This is achieved by applying state feedback controller to the system. Also, self balancing is expected in unmanned mode. An observer is designed and all estimated states are fed back to the system. Data of real system response is discussed in this chapter.

Open loop system response is observed in real time before designing controller. The system is releasing from its upright position. The response is shown in Figure 6.1. Pitch angle is demonstrated in radian. It falls down until anyone holds it. Hence, designing a controller is required.

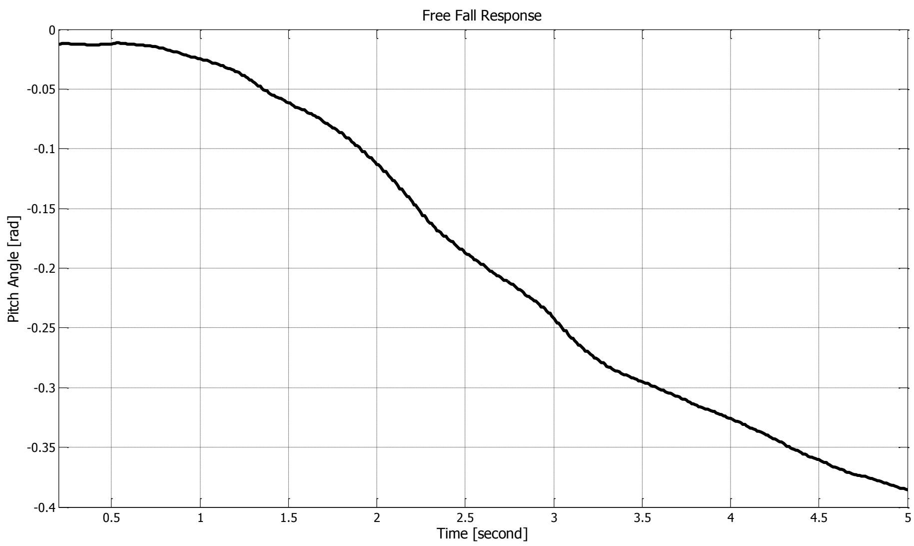

Figure 6.1 - Free Fall Response of 2TEA

6.1 Transporter Mode

State feedback controller maintains the stability in this mode. Pitch angle and angular rate about pitch axis are the only states which are measured, so these states feed back to the system. State feedback control model for real system is constructed in Matlab/Simulink using xPC Target, as in Figure 6.2. In this model, 2TEA includes IMU and controller input triggers the motor controller via analog output of single board computer.

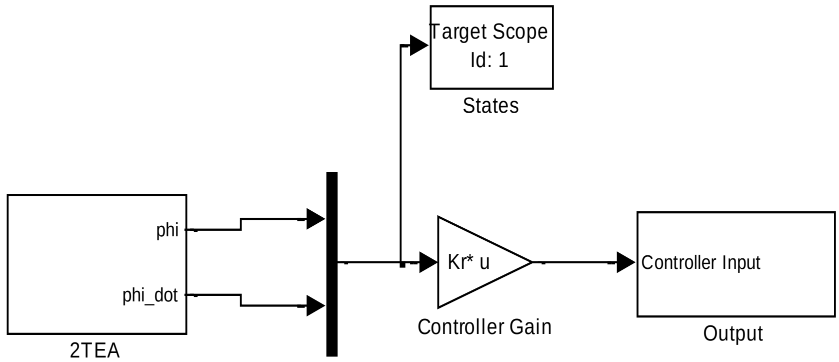

Figure 6.2 - State Feedback Model on Actual System

Controller gains using in simulation couldn't maintain the stabilization on real system. This is because of ignoring some parameters in real world while deriving mathematical model. Also, limited range of system input is a constraint to determine controller gain. New controller gain which is in Equation 6.1 was determined by tuning in real time and was performed on real system.

$$K_r = [-45 \quad -3] \tag{6.1}$$

The driver during this experiment was 1.8 m height and 70 kg. weight. Driver leaned towards forward and backward, and the system moved. The system was able to carry the driver without moving when the driver did not lean. Pitch angle during the motion is depicted in Figure 6.3. It is seen that angle stays constant at around 0.005 rad instead of 0 rad. because of sensor bias.

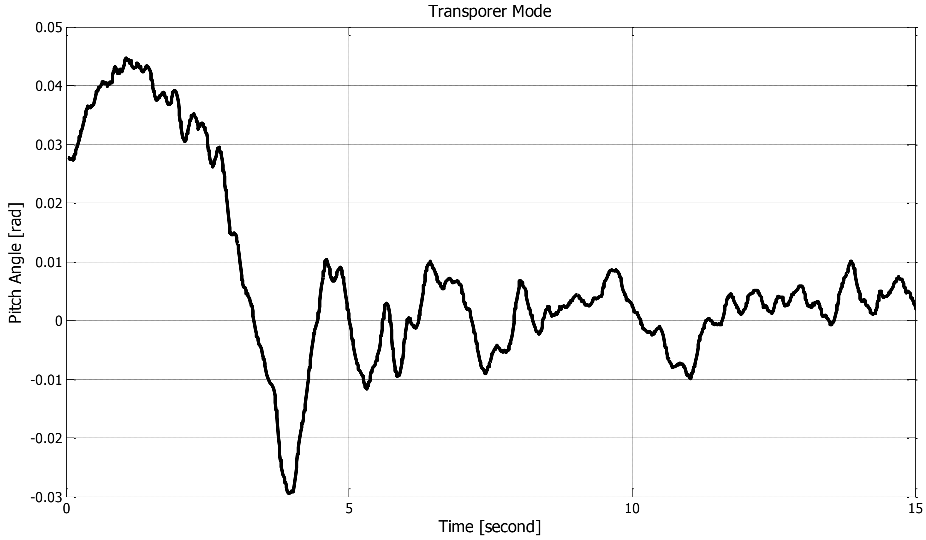

Figure 6.3 - Transporter Mode Response

6.2 Unmanned Mode

Velocity feedback is employed on real system in unmanned mode. This state is estimated by observing phi and phi_dot via IMU. It is assumed that the only sensor is IMU. Thus, all states are estimated and generate controller input. Regulation of pitch angle and angular rate satisfy desired performance.

Before applying state feedback controller with observer, state feedback controller design on transporter mode was implemented on unmanned mode. Although the states are just considered as phi and phi_dot, system response was not insufficient. Pitch response is shown in Figure 6.4.

Controller gain is tuned as below. In addition, observer gain is derived from the desired observer poles which are chosen four times further than controller poles.

$$K_r = [-0.8\ -39\ -5] \tag{6.2}$$

$$K_{est} = \begin{bmatrix} 1.2139 & 17.8399 \\ 8 & 1 \\ 13.5966 & 13.0741 \end{bmatrix} \tag{6.3}$$

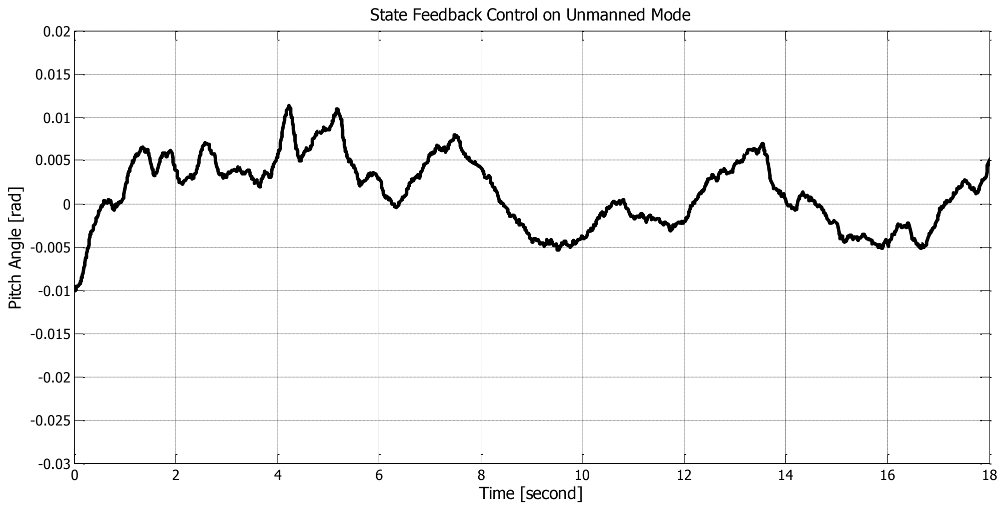

Figure 6.4 - State Feedback Control on Unmanned Mode

Figure 6.5 depicts Simulink model of the system in real time. Observer subsystem gets the states pitch angle and angular rate as input, and gives estimated states as output. Controller input is both system input and observer input. Response of pitch angle is shown in Figure 6.6.

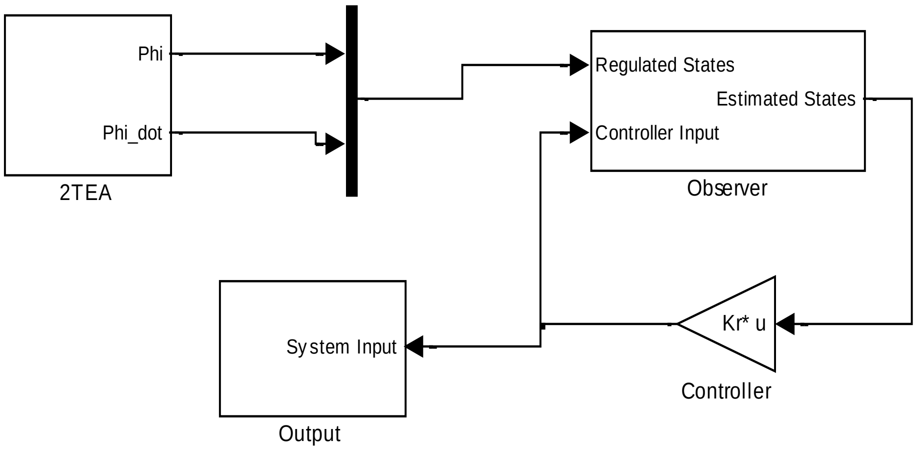

Figure 6.5 - State Feedback with Observer Model on Actual System

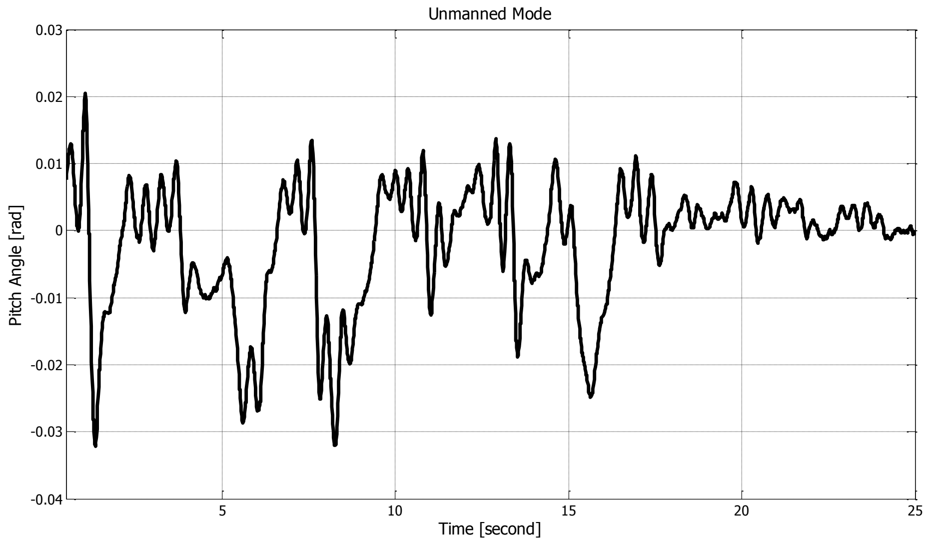

Figure 6.6 - Unmanned Mode Response

However, an incremental encoder is integrated to the system. Thus, all states and estimated states can be compared. The comparison between estimated states and measured states in real time are depicted in figures below. It is clear that the error is not much and reasonable estimation is satisfied. Also, estimated velocity signal attenuates unwanted peaks in measured velocity signals.

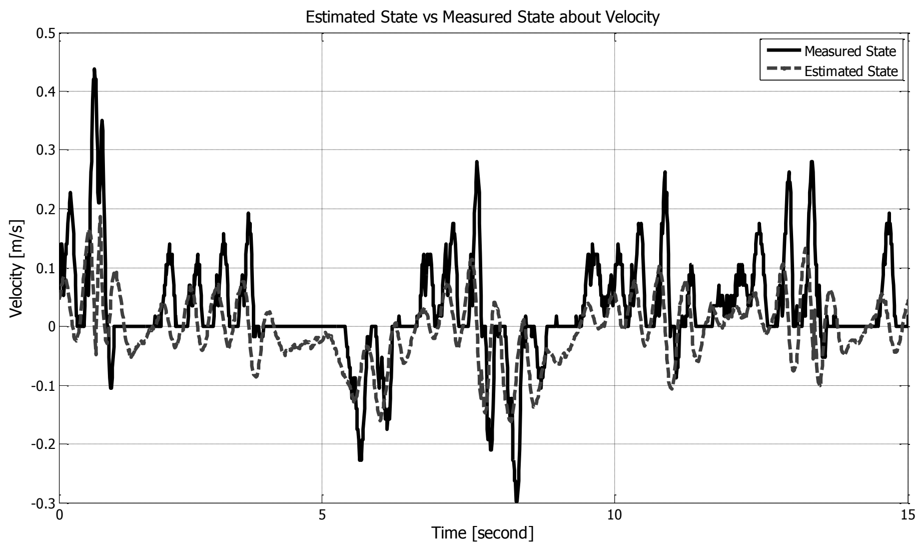

Figure 6.7 - Estimated States vs. Measured States about Velocity

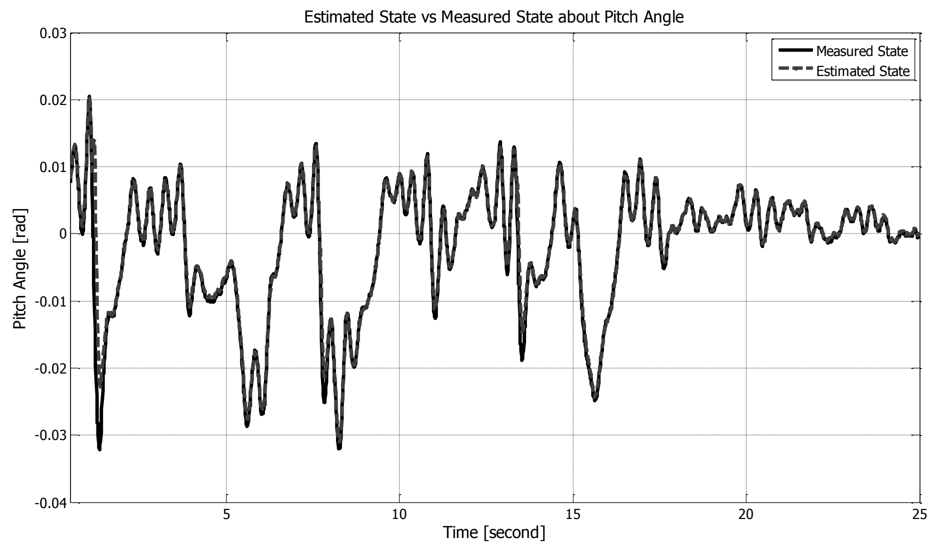

Figure 6.8 - Estimated States vs. Measured States about Pitch Angle

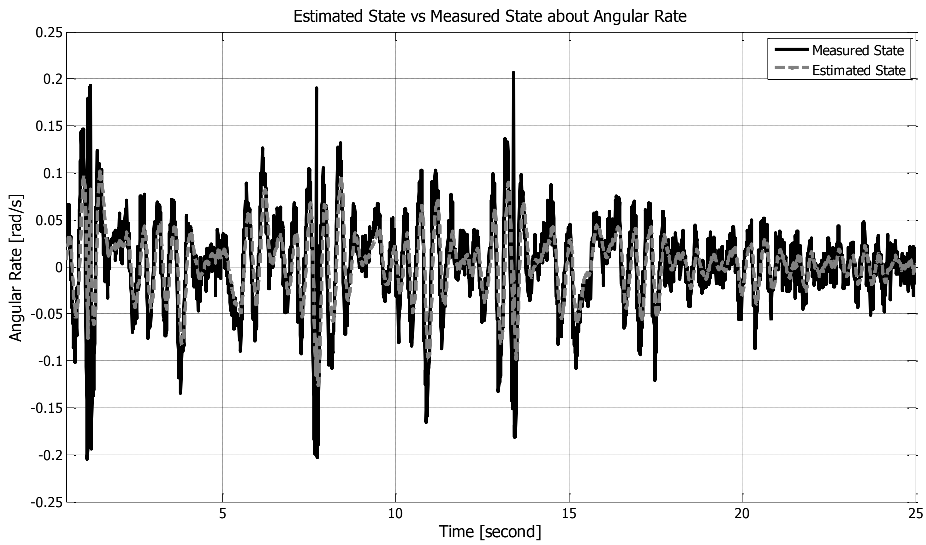

Figure 6.9 - Estimated States vs. Measured States about Angular Rate

CHAPTER 7

CONCLUSION AND DISCUSSION

2TEA is the study on a two-wheeled self balancing electric transporter which can operate in manned and unmanned mode. It was constructed as a term project in Mechatronics in Automotive Engineering course in 2009.

As part of this thesis, physical system is improved. A load carrier mechanism is mounted in front of the system. This is the first step to transform 2TEA into an assistant robot. Also, electrical construction is rearranged and components are placed more appropriately. Sensor set is improved by integrating an encoder on the shaft of the motor.

In addition, system parameters are derived experimentally. Motor parameters and moment of inertia of the system are unknown parameters which are used in deriving mathematical model. Therefore, moment of inertia tests and motor tests are performed to determine these parameters. Controlling pitch dynamic is the essential in 2TEA to maintain stabilization, so pitch and translational dynamics are studied in mathematical model. Equations of motions are derived from Newton's Law of Motion.

State feedback control is implemented to the system on manned mode. The system is able to operate in accordance with driver's lean. Designed regulator in simulation achieves to regulate pitch angle and angular rate though driver's disturbance in the model. Also, this regulator satisfies stabilization of the system with various sized driver. Velocity feedback is implemented by designing an observer. Pitch angle and angular rate are observed to estimate all states. This method is necessary in unmanned mode. It is proven that estimated states are close enough to actual states.

However, some oscillation occurs in real time applications. The system achieves to run in both autonomous and semi-autonomous mode. Stabilization is reasonable, but little effects of sensor bias and oscillation are seen on the response. Velocity reference tracking in unmanned mode is out of scope of this thesis and still in progress.

In the future studies, sensor set should be improved by adding GPS for unmanned mode. Thus, it will be able to navigate in unmanned mode. Also, load will be considered as disturbance and disturbance rejection will be studied on this mode. Thus, a robust controller such as H_{inf} which is based on optimization technique will be designed. In addition, robust controller satisfies better performance in manned mode in order to carry different sized drivers.

In addition, 2TEA is a platform which can be used in wide range of application area. This platform can be combined with a robot arm in order to create an assistant robot or, it can be actuator of a humanoid robot in the future studies.

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APPENDIX A

```
%%%%Transporter Mode Simulation (State Feedback)%%%%

clear all; clc

syms x dx ddx th dth ddth Va f1 f2 f3 f4 L r M m g Kt Ke Ib Iw b R n
Kp Kd Ki s

% Equations from free body diagram and implementation of Newton's
2nd law

% EOM 1.
E1=((Iw/r)-(0.5*M*r)-
m*r)*ddx+((b/r)+(n*n*Kt*Ke/(R*r)))*dx+(0.5*M*r*L*cos(th))*ddth-
(0.5*M*r*L*sin(th))*dth*dth-((n*Kt*Ke+b*R)/R)*dth-((n*Kt)/R)*Va;

% EOM 2.
E2
=(Ib+M*((L*cos(th))^2+(L*sin(th))^2))*ddth+((2*n*Kt*Ke+2*b*R)/R)*dth
-(M*L*cos(th))*ddx-((2*b/r)+(2*n*n*Kt*Ke/(r*R)))*dx+((2*n*Kt)/R)*Va-
(M*g*L*sin(th));


S=solve(E1,E2,ddth,ddx);

(collect(S.ddx,[x,dx,th,dth,Va]));

(collect(S.ddth,[x,dx,th,dth,Va]));

f1=dx;
f2=S.ddx;
f3=dth;
f4=S.ddth;

x = [x dx th dth];
u=[Va];
A=jacobian([f1;f2;f3;f4],x);
B=jacobian([f1;f2;f3;f4],u);

%Driver with 1.80m, 70 kg.

L=0.74; %__with driver (m)
r=0.32 ; % wheel radius (m)
M=102 ; % Mass of Body with driver (kg)
m=2.08;%Mass of Wheel (kg)
g= 9.8 ;%Gravity (m/s^2)
Kt=0.05 ;%Torque Constant of Motor (N/A)
Ke=0.05 ;%Back EMF Constant (Velocity constant) (Volt per rad/s)
Idriver=(1/12)*70*(((1.9)^2)+((0.3)^2));
```

```
Ib=2.2686+Idriver; %Inertia of body with driver,
Iw=0.0165; %Inertia of wheel (kg.m^2)
b=0.5; % viscous friction (0.5_eski, hesaplanan 0.05)
R=1.4 ; %Resistor of motor
n=28.7 ; % Motor Gear Ratio

%Linearization around 0 tilt angle
x=0; %Initial value
dx=0; %Initial value
th=0; %Initial value
dth=0; %Initial value

A=double(subs(A
B=double(subs(B));

C=[0 0 1 0;0 0 0 1];
rank(obsv(A,C));
rank(ctrb(A,B));
%
*********************************************************************
**
Bd=[[0;0;0;1] B]; %% disturbance fi_dot

% Reduced dynamics with translational vel., pitch angle and pitch
vel.
Ar=A(2:4,2:4);
Br=B(2:4);
Brd=Bd(2:4,:); %% two input: one is disturbance, the other is
controller input
Cr=[1 0 0;0 1 0;0 0 1];
Dr=zeros(3,2);
rank(obsv(Ar,Cr));
rank(ctrb(Ar,Brd));
%
*********************************************************************
**
eig(Ar)

%state feedback with desired poles

des_p=[-2+4*i; -2-4*i; -10]; %controller design
Kr=place(Ar,Br,des_p)
Kr=[Kr(2) Kr(3)]

%Modify the matrices in model with respect to Kr=[Kr(2) Kr(3)]
Arx=Ar(2:3,2:3);
Brdx=[Brd(2,:);Brd(3,:)];
Crx=Cr(2:3,2:3);
eig(Arx)

%%%%Unmanned Mode Simulation (Observer)%%%%

clear all; clc

syms x dx ddx th dth ddth Va f1 f2 f3 f4 L r M m g Kt Ke Ib Iw b R n
Kp Kd Ki s

% Equations from free body diagram and implementation of Newton's
2nd law
```

```
% EOM 1.
E1=((Iw/r)-(0.5*M*r)-
m*r)*ddx+((b/r)+(n*n*Kt*Ke/(R*r)))*dx+(0.5*M*r*L*cos(th))*ddth-
(0.5*M*r*L*sin(th))*dth*dth-((n*Kt*Ke+b*R)/R)*dth-((n*Kt)/R)*Va;

% EOM 2.
E2
=(Ib+M*((L*cos(th))^2+(L*sin(th))^2))*ddth+((2*n*Kt*Ke+2*b*R)/R)*dth
-(M*L*cos(th))*ddx-((2*b/r)+(2*n*n*Kt*Ke/(r*R)))*dx+((2*n*Kt)/R)*Va-
(M*g*L*sin(th));


S=solve(E1,E2,ddth,ddx);

(collect(S.ddx,[x,dx,th,dth,Va]));

(collect(S.ddth,[x,dx,th,dth,Va]));

f1=dx;
f2=S.ddx;
f3=dth;
f4=S.ddth;

x = [x dx th dth];
u=[Va];
A=jacobian([f1;f2;f3;f4],x);
B=jacobian([f1;f2;f3;f4],u);

L=0.1; %__without driver
r=0.32 ; % wheel radius (m)
M=31.95; %Mass of Body__without driver (kg)
m=2.08;%Mass of Wheel (kg)
g= 9.8 ;%Gravity (m/s^2)
Kt=0.05 ;%Torque Constant of Motor (N/A)
Ke=0.05 ;%Back EMF Constant (Velocity constant) (Volt per rad/s)
Ib=2.2686; %Inertia of body without driver,
Iw=0.0165; %Inertia of wheel (kg.m^2)
b=0.5; % viscous friction (0.5_eski, hesaplanan 0.05)
R=1.4 ; %Resistor of motor
n=28.7 ; % Motor Gear Ratio

%Linearization around 0 tilt angle
x=0; %Initial value
dx=0; %Initial value
th=0; %Initial value
dth=0; %Initial value

A=double(subs(A));
B=double(subs(B));

C=[0 0 1 0;0 0 0 1];
rank(obsv(A,C));

%
*******************************************************************
**
% Reduced dynamics with translational vel., pitch angle and pitch
vel.
```

```
Ar=A(2:4,2:4);
Br=B(2:4);
Cr=[1 0 0;0 1 0;0 0 1];
Dr=zeros(3,2);
rank(obsv(Ar,Cr));
rank(ctrb(Ar,Br));
%
********************************************************************
**
eig(Ar)

%%desired poles for full state observer %%%%

des_p=[-1.5+2*i; -1.5-2*i; -2]; %des_o=2*des_p

Kr=place(Ar,Br,des_p)

%observer design
Crd=[Cr(2,:);Cr(3,:)];
des_o=2*des_p; %% observer is two times faster than plant
Kest=place(Ar',Crd',des_o)'; %%%observer gain ___ only phi and dphi
observed and all est. states feedback
```